Schedule for the International CIMPA School, 2026

Topological and Extremal Properties of Combinatorial Structures

6-18 July 2026

Time table

Date	Time	Activity	Instructor/Details
Mon, Week 1	08:00 - 10:00	**Opening Ceremony**	
	10:00 - 10:30	Coffee Break	
	10:30 - 12:30	**Lecture 1**	Prof. B. Servatius
	12:30 - 14:00	Lunch	
	14:00 - 16:00	**Lecture 2 (Part 1)**	Prof. F. Bassino
	16:00 - 16:30	Coffee Break	
	16:30 - 18:00	**Lecture 1**	Prof. B. Servatius
	18:00 - 20:00	Dinner	
Tues, Week 1	08:00 - 10:00	**Lecture 2 (Part 1)**	Prof. F. Bassino
	10:00 - 10:30	Coffee Break	
	10:30 - 12:30	**Lecture 2 (Part 2)**	Prof. S. Wagner
	12:00 - 14:00	Lunch	
	14:00 - 16:00	Lecture 1 (CANCELLED) **Lecture 1 Part 2**	Prof. S. Wagner
	16:00 - 16:30	Coffee Break	
	16:30 - 18:00	**Round Table on Gender Issues in Mathematics**	
	18:00 - 20:00	**Welcome Dinner**	
Wed, Week 1	08:00 - 10:00	**Lecture 2 (Part 2)**	Prof. S. Wagner
	10:00 - 10:30	Coffee Break	
	10:30 - 12:30	**Lecture 2 (Part 1)**	Prof. F. Bassino
	12:30 - 14:00	Lunch	
	14:00 - 16:00	**Lecture 2 (Part 2)**	Prof. S. Wagner
	16:00 - 16:30	Coffee Break	
	16:30 - 18:30	**Parallel: Group projects presentation**	
	18:30 - 20:00	Dinner	
Thurs, Week 1	08:00 - 10:00	**Lecture 4**	Dr. A. Dossou-Olory
	10:00 - 10:30	Coffee Break	
	10:30 - 12:30	**Lecture 3**	Dr. V. Misanantenaina Rakotonarivo
	12:30 - 14:00	Lunch	
	14:00 - 16:00	**Lecture 3**	Dr. V. Rakotonarivo Misanantenaina
	16:00 - 16:30	Coffee Break	
	16:30 - 18:30	**Parallel: Group projects**	
	18:00 - 20:00	Dinner	
Frid, Week 1	08:00 - 10:00	**Lecture 3**	Dr. V. Rakotonarivo Misanantenaina
	10:00 - 10:30	Coffee Break	
	10:30 - 12:10	**Lecture 4**	Dr. A. Dossou-Olory
	12:10 - 12:30	**CIMPA: Actions and Initiatives (Meeting)**	Dr. N. Bédaride (CIMPA representative)
	12:30 - 14:00	Lunch	
	14:00 - 16:00	**Lecture 3**	Dr. V. Misanantenaina Rakotonarivo
	16:00 - 16:30	Coffee Break	
	16:30 - 18:30	**Parallel: Group projects**	

Date	Time	Activity	Instructor/Details
	18:00 - 20:00	Dinner	
Sat, Week 1	All Day	Break Day / **Excursion**	
Mon, Week 2	08:00 - 10:00	**Lecture 1**	Pr. B. Servatius
	10:00 - 10:30	Coffee Break	
	10:30 - 12:30	**Lecture 5 (Part 1)**	Dr. K. Dadedzi
	12:30 - 14:00	Coffee Break	
	14:00 - 16:00	**Lecture 6**	Dr. R. Avohou
	16:00 - 16:30	Coffee Break	
	16:30 - 18:00	**Open forum: Mathematics in Africa; Number and Pattern in African Cultures**	
	18:00 - 20:00	Dinner	
Tues, Week 2	08:00 - 10:00	**Lecture 5 (Part 1)**	Dr. K. Dadedzi
	10:00 - 10:30	Coffee Break	
	10:30 - 12:30	**Lecture 6**	Dr. R. Avohou
	12:30 - 14:00	Lunch	
	14:00 - 16:00	**Lecture 4**	Dr A. Dossou-Olory
	16:00 - 16:30	Coffee Break	
	16:30 - 18:30	**Parallel: Group projects**	
	18:00 - 20:00	Dinner	
Wed, Week 2	08:00 - 10:00	**Lecture 5 (Part 1)**	Dr. K. Dadedzi
	10:00 - 10:30	Coffee Break	
	10:30 - 12:30	**Lecture 5 (part 2)**	Prof. E. Andriantiana
	12:30 - 14:00	Lunch	
	14:00 - 16:00	**Lecture 6**	Dr. R. Avohou
	16:00 - 16:30	Coffee Break	
	16:30 - 18:00	**Lecture 7**	Prof. J. Ben Geloun
	18:00 - 20:00	Dinner	
Thurs, Week 2	08:00 - 10:00	**Lecture 5 (Part 2)**	Prof. E. Andriantiana
	10:00 - 10:30	Coffee Break	
	10:30 - 12:30	**Lecture 7**	Prof. J. Ben Geloun
	12:30 - 14:00	Lunch	
	14:00 - 16:00	**Lecture 5 (Part 2)**	Prof. E. Andriantiana
	16:00 - 16:30	Coffee Break	
	16:30 - 18:00	**Parallel:** **Group projects**	
	18:00 - 20:00	**Farewell Dinner**	
Frid, Week 2	08:00 - 10:00	**Lecture 7**	Prof. J. Ben Geloun
	10:00 - 10:30	Coffee Break	
	10:20 - 12:20	**Presentation of the group projects**	
	12:30 - 14:00	Lunch	
	14:00 - 14:45	**Presentation of the group projects**	
	14:45 - 15:00	Break	
	15:00 - 16:00	**Research Presentations (talks)**	
	16:00 - 16:30	Coffee Break	
	16:30 - 18:00	**Research Presentations (talks)**	
	18:00 - 20:00	Dinner	
Sat, Week 2	All Day	Break Day	Departure

Lecture syllabi

Lecture 1: Introduction to Combinatorics and Graph Theory

Instructor: Prof. Brigitte Servatius
Duration: 4 lectures (2 hours each)
Reference: *Martin Milanic, Brigitte Servatius and Herman Servatius. Discrete mathematics with logic. London; San Diego; Academic press, 2024.*

Course description

This short course introduces some of the fundamental concepts of discrete mathematics, including finite sets, counting techniques, mathematical induction, coding and cryptography, and graph theory. The emphasis is on developing logical reasoning and problem-solving skills through concrete examples and applications drawn from computer science, communication networks, and combinatorics.

Learning objectives

By the end of the course, participants will be able to:

- Work confidently with finite sets and basic combinatorial structures;
- Apply elementary counting techniques to solve discrete problems;
- Use mathematical induction to prove statements involving integers and recursive structures;
- Understand the mathematical principles underlying codes and ciphers;
- Describe and analyze basic graph-theoretic structures, including trees.

Schedule

Chap. 1: Basic Combinatorial Objects and Finite Sets

- Sets and set operations
- Finite sets and cardinality
- Counting principles
- Applications and examples

Chap. 2: Mathematical Induction

- Recursive definitions
- Principle of induction
- Strong induction
- Applications to combinatorics and number theory

Chap. 3: Codes and Ciphers

- Introduction to coding theory
- Classical cryptographic methods
- Error detection and correction
- Applications in secure communication

Chap. 4: Graphs and Trees

- Basic graph concepts
- Paths, cycles, and connectivity
- Trees and their properties
- Applications to networks and algorithms

Teaching Method

Each lecture combines theoretical presentations, worked examples, and problem-solving sessions designed to encourage active participation and mathematical exploration.

Lecture 2: Enumerative Combinatorics

Lecture 2 Part 1: Enumerative Combinatorics: Exact Enumeration, Formal Power Series, and Symbolic Methods

Instructor: Frédérique Bassino
Duration: 4 lectures (2 hours each, 8 hours total)
References:

1. M. Bóna, *A Walk Through Combinatorics: An Introduction to Enumeration and Graph Theory*, 4th ed., Springer, 2016.
2. H. S. Wilf, *Generatingfunctionology*, 3rd ed., A K Peters, 2005.

Course description

Enumerative combinatorics is concerned with the counting of discrete structures and the development of systematic methods for deriving counting formulas. This short course introduces the fundamental tools of exact enumeration, with a particular emphasis on formal power series and symbolic methods. These techniques provide a powerful framework for translating combinatorial constructions into algebraic equations whose solutions yield counting sequences and generating functions.

Through a variety of classical examples, participants will learn how combinatorial classes can be specified, manipulated, and enumerated using symbolic constructions and generating functions. The course serves as an introduction to modern combinatorial enumeration and provides the foundations for more advanced topics in analytic combinatorics.

Learning objectives

By the end of the course, participants will be able to:

- Describe combinatorial classes using symbolic specifications;
- Apply basic counting principles to enumerate discrete structures;
- Manipulate formal power series and interpret their coefficients combinatorially;
- Construct and use generating functions for exact enumeration;
- Translate recursive and combinatorial constructions into functional equations;
- Solve classical enumeration problems using symbolic methods.

Schedule

Chap. 1: Introduction to Exact Enumeration

- Basic counting principles
- Combinatorial classes
- Classical counting problems
- Exact enumeration of simple combinatorial objects

Chap. 2: Generating Functions and Symbolic Methods

- Formal power series
- Ordinary/exponential generating functions

- Algebraic operations on generating functions
- Symbolic method : Translation rules into generating functions
- Recursive combinatorial structures

Chap. 3: Applications and Advanced Examples

- Functional equations arising from combinatorial specifications
- Enumeration of trees and related structures
- Classical examples from combinatorics
- Perspectives toward analytic combinatorics

Teaching Method

The course combines lectures, worked examples, and problem-solving sessions. Participants will gain hands-on experience in constructing generating functions and applying symbolic methods to solve enumeration problems. Special emphasis will be placed on developing intuition through concrete combinatorial examples and interactive discussions.

Lecture 2 Part 2: Analytic Methods in Enumerative Combinatorics

Instructor: Prof. Stephan Wagner
Duration: 5 lectures (2 hours each, 10 hours total)
References:

- Flajolet, P. and Sedgewick, R., *Analytic Combinatorics*, Cambridge University Press, Cambridge, 2009.
- Melczer, S., An invitation to analytic combinatorics—from one to several variables. Springer, Cham, 2021.

Course description

Generating functions are among the most powerful tools in modern enumerative combinatorics. They not only encode counting sequences in a compact algebraic form but also provide deep insights into the asymptotic behaviour of combinatorial structures through their analytic properties. This course introduces the fundamental principles of asymptotic enumeration, focusing on the relationship between generating functions, their singularities, and the growth of their coefficients.

Participants will learn how analytic techniques can be used to derive asymptotic estimates for counting sequences and how these methods apply to a wide range of combinatorial structures. Particular emphasis will be placed on singularity analysis, one of the central tools of analytic combinatorics.

Learning objectives

By the end of the course, participants will be able to:

- Understand the connection between generating functions and counting sequences;
- Relate the asymptotic growth of coefficients to the analytic properties of generating functions;
- Analyze rational and meromorphic generating functions;
- Apply singularity analysis to derive asymptotic estimates;
- Use transfer theorems in asymptotic enumeration;
- Apply analytic methods to a variety of combinatorial examples.

Schedule

Chap. 1: Generating Functions and Asymptotics

- Basic principles of asymptotic analysis
- Generating functions as tools for enumeration
- Relationship between coefficient growth and singularities
- Illustrative combinatorial examples

Learning objectives:

- Discuss the basic principles of asymptotic analysis;
- Explain the connection between the growth of coefficients and the singularities of a generating function;
- Study representative examples arising in combinatorics.

Chap. 2: Rational and Meromorphic Functions

- Rational generating functions
- Meromorphic generating functions
- Pole analysis and asymptotic coefficient estimates
- Combinatorial applications

Learning objectives:

- Understand how asymptotic information can be extracted from rational and meromorphic generating functions;
- Derive asymptotic formulas from singularities;
- Examine combinatorial structures whose generating functions are rational or meromorphic.

Chap. 3: Singularity Analysis

- Principles of singularity analysis
- Local expansions around dominant singularities
- Transfer theorems
- Applications to combinatorial enumeration

Learning objectives:

- Explain the fundamental ideas of singularity analysis;
- Understand the proof and significance of the main transfer theorem;
- Apply singularity analysis to relevant combinatorial examples.

Chap. 4: Tutorial - Asymptotic Analysis of Generating Functions

- Guided exercises
- Application of singularity analysis techniques
- Worked combinatorial examples

Learning objectives:

- Practise the methods introduced in the lectures;
- Develop confidence in asymptotic computations;
- Explore additional motivating examples from combinatorics.

Teaching Method

The course combines theoretical lectures with interactive tutorial sessions. Emphasis is placed on worked examples and problem-solving activities, allowing participants to gain practical experience with asymptotic methods and singularity analysis. The tutorials are designed to reinforce the theoretical material and provide exposure to a broad range of combinatorial applications.

Lecture 3: Graph Algorithms and Programming

Instructor: Dr. Valisoa Razanajatovo Misanantenaina Rakotonarivo
Duration: 4 lectures (2 hours each, 8 hours total)
Software: Python and the `NetworkX` library
Reference:

1. A. A. Hagberg, D. A. Schult, and P. J. Swart, *Exploring Network Structure, Dynamics, and Function using NetworkX*, Proceedings of the 7th Python in Science Conference, 2008.

2. A. M. J. van Steen, *Graph Theory and Complex Networks: An Introduction*, Maarten van Steen, 2010.

3. J. Kleinberg, and E. Tardos, *Algorithm Design*, Addison-Wesley, 2005.

Course description

Graphs provide a natural mathematical framework for modeling networks, relationships, and interactions arising in computer science, engineering, biology, social sciences, and many other disciplines. This hands-on course introduces participants to graph algorithms and computational graph theory using Python and the `NetworkX` library, one of the most widely used tools for network analysis.

The course combines theoretical concepts with practical programming exercises. Participants will learn how to construct, manipulate, visualize, and analyze graphs, as well as how to implement and apply classical graph algorithms. Particular emphasis will be placed on distance-based graph parameters and real-world applications of graph analysis.

Learning objectives

By the end of the course, participants will be able to:

- Create and manipulate graphs using the `NetworkX` library;
- Visualize graph structures and network data;
- Implement and analyze classical graph algorithms;
- Compute distance-based graph parameters and centrality measures;
- Apply graph-theoretic methods to practical problems;
- Develop Python programs for network analysis and graph exploration.

Schedule

Chap. 1: Introduction to NetworkX and Graph Programming

- Introduction to Python for graph analysis
- Overview of the `NetworkX` library
- Graph creation and manipulation
- Working with nodes, edges, and graph attributes
- Graph visualization and plotting

Learning objectives:

- Become familiar with the `NetworkX` environment;
- Create and modify graph structures programmatically;
- Visualize graphs and interpret their representations.

Chap. 2: Distance-Based Graph Algorithms and Network Measures

- Review of fundamental graph concepts
- Paths, distances, and connectivity
- Shortest-path algorithms
- Centrality measures and network importance
- Applications to real-world networks

Learning objectives:

- Understand key distance-based graph parameters;
- Implement shortest-path computations;
- Analyze networks using centrality and related measures.

Chap. 3: Advanced Applications and Network Analysis

- Exploration of complex network datasets
- Additional graph metrics and structural properties
- Comparative analysis of networks
- Practical case studies using `NetworkX`

Learning objectives:

- Apply graph algorithms to realistic datasets;
- Interpret the results of network analyses;
- Gain experience with computational graph investigations.

Chap. 4: Open-Ended Problem Solving Workshop

- Guided graph-theoretic projects
- Problem-solving using Python and `NetworkX`
- Design and implementation of computational solutions
- Discussion of results and alternative approaches

Learning objectives:

- Consolidate the concepts and techniques introduced throughout the course;
- Develop independent problem-solving skills;
- Apply graph algorithms and programming tools to open-ended challenges.

Teaching Method

The course is strongly practice-oriented and combines short theoretical presentations with live coding demonstrations, guided programming exercises, and problem-solving activities. Participants are encouraged to actively experiment with graph algorithms using Python and `NetworkX`, gaining hands-on experience in computational graph theory and network analysis.

Lecture 4: Extremal Graph Theory: Generalities

Instructor: Dr. Audace A. V. Dossou-Olory
Duration: 3 lectures (2 hours each)
Reference:

- N. Alon, M. Krivelevich (2008), *Extremal and Probabilistic Combinatorics*, pp.562–575
- B. Bollobás (2004), *Extremal Graph Theory.*
- B. Bollobás (1998), *Modern Graph Theory*, pp.103–144
- R. Wenger (1991), *Extremal graphs with no C4, C6 and C1*0, p.113–116.

Course description

In this course, we will introduce basic instances of extremal graph theory statement, and study the most classical type of extremal question in graphs, where we impose a natural property of graphs (some constraint on the graph) and ask how many edges it can have subject to that constraint. Our main focus will be on forbidding some fixed subgraph. This notion was first studied by Mantel in 1907. We will establish both lower and upper (exact or asymptotic) bounds on the maximum number of edges of a n-vertex graph that does not contain a given fixed graph as a subgraph. In particular, we will explore properties of Turán graph, the Erdös-Stone-Simonovits theorem, the Kövari-Sós-Turán theorem, Klein's theorem and Brown theorem. We will conclude the lectures with the exploration of Ramsey-type problems in a way of combinatorial exercices.

Learning objectives

By the end of the course, participants will be able to:

- Formulate extremal graph theory statements;
- Understand the fundamental theorem of extremal graph theory;
- Distinguish between exact and asymptotic extremal numbers
- Use natural properties of graphs to analyze extremal graph structures;
- Apply fundamental results from extremal graph theory;
- Appreciate connections between number theory, counting principles in combinatorics, algebra, and graph theory.

Schedule

Chap. 1: Foundations of Extremal Graph Theory

- Extremal graph problems
- Extremal functions
- Classical examples and motivations

Learning objectives:

- Understand the nature of extremal graph problems;
- Formulate extremal questions in graph theory;
- Apply basic extremal techniques.

Chap. 2: Turán's Theorem and Extremal Results

- Turán graphs
- Turán's Theorem

- Generalizations and applications

Learning objectives:

- Master one of the central results of extremal graph theory;
- Apply Turán-type arguments to graph problems;
- Explore applications and open problems.

Chap. 3: Ramsey Theory and Extremal Problems

- Ramsey numbers
- Fundamental Ramsey-type results
- Applications and open problems

Learning objectives:

- Understand the basic principles of Ramsey theory;
- Analyze classical Ramsey problems;
- Appreciate the role of extremal methods in modern graph theory.

Teaching Method

Each lecture combines active participation and mathematical exploration. There will be no advanced proof strategies as all materials will gradually be built from basic graph instances.

Lecture 5: Extremal Aspects in Spectral Graph Theory

Instructors:

- Prof. Eric Andriantiana
- Dr. Kenneth Dadedzi

Duration: 6 lectures (2 hours each)

Reference:

- Biggs, N. L. Algebraic Graph Theory. 2nd ed. Cambridge Mathematical Library. Cambridge: Cambridge University Press, 1993.
- Cvetković, D. (Ed.). (2008). Zbornik radova (Vol. 13(21)). Beograd: Matematički institut SANU.
- EODA, Energy, Hosoya index and Merrifield-Simmons index of trees with prescribed degree sequence, Discrete Appl. Math. 161 (2013) 724-741.
- EODA and S. Wagner, Spectral moment of trees with given degree sequence, Linear Algebra Appl. 439 (2013) 3980-4002.
- Cvetković, Dragoš M., Peter Rowlinson, and Slobodan Simić. An introduction to the theory of graph spectra. Vol. 75. Cambridge: Cambridge university press, 1945.

Course description

This advanced course explores several modern directions in graph theory, bringing together combinatorial, spectral, and extremal perspectives. Participants will be introduced to the theory of graph spectra. Moreover, they will study graph energy and its connections with matchings and independent sets, spectral moments and spectral invariants such as the spectral radius and Estrada index, and fundamental results in extremal graph theory. These topics play an important role in contemporary graph theory and have applications in network science, computer science, chemistry, and combinatorial optimization.

The course combines theoretical developments with examples and problem-solving sessions, enabling participants to acquire both a solid theoretical understanding and practical techniques for tackling advanced graph-theoretic problems.

Part I: Spectral Graph Theory: Spectral-Based Graph Invariants (Dr. K. Dadedzi)

Chap. 1: Graph Spectra

- Matrices associated with graphs, namely, the incidence matrix, the adjacency matrix, the distance matrix, the Laplacian matrix, the Signless Laplacian matrix, the Seidel matrix, the ancestral matrix, the level matrix, and the terminal distance matrix.
- The spectrum of the adjacency matrix.

Learning objectives:

- Familiar with several matrices associated with graphs.
- Understand how to compute the adjacency spectrum of some classes of graphs.

Chap. 2: Graph Operations and Spectral techniques

- Graph operations that affect the spectrum of graphs.
- Characterizations of graphs by their spectra.
- Graph spectrum and graph structure.

Learning objectives:

- Understand the spectrum of some graphs.
- Apply some graph techniques to compute or estimate the spectrum of some graphs.
- Understand the tools to bound spectral-based invariants.

Chap. 3: Spectral-Based invariants

- Definition and properties of the spectral radius of graphs.
- Definition and properties of the energy of graphs.
- Definition of spectral moments.
- Definition of walks and their connection to spectral moments.
- Relationships between the spectral moments of a graph and the Estrada index.

Learning objectives:

- Understand the relationship between the spectral radius, graph energy and other graph parameters and invariants.
- Understand the connections between the number of walks, spectral moments, and the Estrada index of a graph.
- Understand the effect of selected graph transformations (in chap 2) on these invariants.

Part II: Selected Applications of Spectral Graph Theory: Energy of graphs, Spectral Moments, Number of Walks and Estrada Index (Prof. E. Andriantiana)

Chap. 4: Spectral Moments and Walks in Graphs

- Spectral moments
- Closed walks and trace methods
- Enumeration of walks through spectral techniques

- Relation to the Estrada index

Learning objectives:

- Understand the relationship between spectral moments and graph structure;
- Count walks using spectral methods;
- Interpret spectral moments combinatorially

Chap. 5: Spectral Radius

- Definition and properties
- Perron–Frobenius theory
- Bounds and extremal spectral problems

Learning objectives:

- Compute and estimate the spectral radius;
- Understand its structural significance;
- Apply spectral radius techniques to graph analysis.

Chap. 6: Energy of Graphs and the number of matchings

- Formulas and more properties of the energy of graphs,
- Relation between the energy of graphs and the number of independent edge subsets.
- Useful tools for the study of Energy of graphs

Learning objectives:

- Understand how to use linear algebra to study the energy of graphs;
- Understand how to use the number of matchings to study the energy of graphs.
- Become familiar with graph structures that attain maximum or minimum energy.

Teaching Method

The course combines theoretical lectures, illustrative examples, and interactive problem-solving sessions. Particular emphasis will be placed on the interplay between combinatorial and spectral techniques, enabling participants to gain a broad perspective on contemporary graph theory and its applications.

Lecture 6: Matroid Theory

Instructor: Dr. Rémi Cocou Avohou
Duration: 3 lectures (2 hours each, 6 hours total)
References:

- **James Oxley** — *Matroid Theory*, Wiley, 1987.
  Matroid Theory, 2nd edition, Oxford Graduate Texts in Mathematics, Oxford University Press, 2011.
- **Neil White** — *Combinatorial Geometries*, Wiley, 1987.
  Theory of Matroids, Encyclopedia of Mathematics and its Applications, Cambridge University Press, 1986.
- **Günter M. Ziegler** — *Lectures on Polytopes*, Wiley, 1987.
  Lectures on Polytopes, Graduate Texts in Mathematics 152, Springer, 1995.

Course description

Matroid theory is a central area of modern combinatorics that provides a unifying framework for studying independence in mathematics. Originating from attempts to abstract the notion of linear independence in vector spaces, matroids connect graph theory, geometry, optimization, coding theory, and polyhedral combinatorics. They provide the theoretical foundation for greedy algorithms and offer powerful tools for understanding combinatorial structures.

This introductory course presents the fundamental concepts of matroid theory, beginning with motivating examples from graph theory and optimization. Participants will learn the basic definitions and properties of matroids, explore representable matroids and their geometric interpretations, and study important combinatorial invariants such as the Tutte polynomial and chromatic polynomial. The course will also introduce geometric and polyhedral aspects of matroids, including matroid base polytopes and Ehrhart polynomials.

Learning objectives

By the end of the course, participants will be able to:

- Understand the role of matroids as abstractions of independence;
- Apply the greedy algorithm in matroid settings;
- Recognize and construct examples of matroids arising from graphs and vector spaces;
- Understand the concepts of bases, circuits, flats, and representability;
- Interpret matroids geometrically and combinatorially;
- Compute and analyze matroid invariants such as the Tutte polynomial;
- Appreciate connections between matroid theory, graph theory, and polyhedral geometry.

Schedule

Chap. 1: Greedy Algorithms and Foundations of Matroid Theory

- Kruskal's algorithm and minimum spanning trees
- Motivation for matroids
- Definitions of matroids
- Independent sets, bases, circuits, and rank
- Examples from graphs and vector spaces

Learning objectives:

- Understand why matroids provide the correct framework for greedy algorithms;
- Master the fundamental definitions and examples of matroids;
- Relate graph-theoretic and linear-algebraic notions of independence.

Chap. 2: Representable Matroids and Geometric Structures

- Representable matroids
- Matroids arising from matrices
- Geometric interpretation of bases and independent sets
- Circuits, broken circuits, and flats
- Lattices of flats and spanning sets

Learning objectives:

- Understand how matroids arise from linear algebra;
- Explore geometric aspects of matroid theory;
- Study the combinatorial significance of circuits, broken circuits, and flats.

Chap. 3: Polynomial and Polyhedral Invariants of Matroids

- Chromatic polynomial via flat decompositions
- The Tutte polynomial of a matroid
- Connections with graph polynomials
- Matroid base polytopes
- Introduction to Ehrhart polynomials
- Applications and research perspectives

Learning objectives:

- Compute and interpret important matroid invariants;
- Understand the relationship between matroids and graph polynomials;
- Explore the polyhedral geometry associated with matroids;
- Gain insight into current research directions in matroid theory.

Teaching Method

The course combines theoretical presentations with examples, guided exercises, and interactive discussions. Particular emphasis will be placed on building intuition through graph-theoretic and geometric examples, while highlighting the deep connections between matroid theory, optimization, algebraic combinatorics, and polyhedral geometry. Participants will be exposed to both classical results and modern developments, providing a foundation for further study and research in the area.

Lecture 7: Topological Graph Theory

Instructor: Prof. Joseph Ben Geloun
Duration: 4 lectures (2 hours each)
References:

- **J. L. Gross & T. W. Tucker** — *Topological Graph Theory*, Wiley, 1987.
  The standard reference: embeddings, rotation systems, voltage graphs, genus.
- **S. Lando & A. Zvonkin** — *Graphs on Surfaces and Their Applications*, Springer, 2004.
  Maps, dessins d'enfants, matrix models, and connections to algebraic geometry.
- **B. Mohar & C. Thomassen** — *Graphs on Surfaces*, Johns Hopkins UP, 2001.
  Rigorous treatment of surface embeddings, Kuratowski, graph minors.
- **R. Diestel** — *Graph Theory*, 5th ed., Springer, 2017. **(free online)**
  Chapters 4 (planar graphs) and 12 (graph minors) are essential.
- **J. Ellis-Monaghan & C. Merino** — *Graph polynomials and their applications I & II*, in *Structural Analysis of Complex Networks*, Birkhäuser, 2011.
  Excellent survey, freely available on arXiv (0803.3079, 0806.4699).

Course description

Topological graph theory studies the interaction between graph theory and topology through the embedding of graphs on surfaces. It provides powerful tools for understanding the structure of graphs by examining their topological properties, such as genus and Euler characteristic, and their associated combinatorial invariants. Among these invariants, the Tutte polynomial occupies a central position, linking graph theory, topology, matroid theory, statistical physics, and knot theory.

This course introduces the fundamental concepts of topological graph theory, including graph embeddings, planar graphs, surface topology, graph minors, and graph polynomials. Particular emphasis will be placed on the Tutte polynomial and its applications to graph coloring, spanning trees, connectivity, and topological properties of embedded graphs. The course concludes with modern perspectives on maps and topological invariants arising from embedded combinatorial structures.

Learning objectives

By the end of the course, participants will be able to:

- Understand graph embeddings on surfaces;
- Distinguish between planar and non-planar graphs;
- Compute and interpret topological invariants such as Euler characteristic and genus;
- Apply classical characterizations of planar graphs;
- Understand the definition and properties of the Tutte polynomial;
- Relate graph polynomials to topological and combinatorial structures;
- Understand the definition of embedded graphs as combinatorial maps;
- Appreciate applications of topological graph theory in mathematics and related disciplines.

Schedule

Chap. 1: Graph Embeddings and Planar Graphs

- Introduction to topological graph theory
- Topological representation of a graph
- Surfaces
- Graph embeddings
- Subdivision and homeomorphism
- Homeomorphism vs. isomorphism
- Cellular embeddings
- Faces
- Planar graphs and plane graphs
- Euler characteristic and genus of a graph
- Consequences of Euler's formula
- Two fundamental non-planar graphs: K_5 and $K_{3,3}$
- Kuratowski's theorem

Learning objectives:

- Understand the notion of graph embeddings;

- Distinguish between abstract and embedded graphs;
- Understand what a planar graph is;
- Understand and apply criteria for determining the non-planarity of graphs.

Chap. 2: Minors, Dual Graphs, Wagner's Theorem

- Graph minors and subdivisions
- Kuratowski's and Wagner's theorems
- Dual graphs

Learning objectives:

- Understand the definition of a graph minor;
- Understand why Wagner's theorem is more general than Kuratowski's theorem;
- Know how to apply these theorems;
- Understand the construction of the dual graph of a planar graph.

Chap. 3: Invariant Polynomials, Ribbon Graphs and Their Representation

- Deletion–contraction rules
- Rank and nullity of a graph
- Definition of the Tutte polynomial
- Recurrence relation for the Tutte polynomial
- Universality
- Ribbon graphs and their representation
- The Bollob'as–Riordan polynomial

Learning objectives:

- Understand the construction of the Tutte polynomial;
- Compute the Tutte polynomial for basic graph families;
- Relate graph polynomials to structural properties of graphs;
- Understand the definition of a ribbon graph;
- Understand the Bollob'as–Riordan polynomial invariant for ribbon graphs.

Teaching Method

The course combines theoretical lectures, visual demonstrations, worked examples, and guided problem-solving sessions. Special emphasis will be placed on geometric intuition, topological reasoning, and the computation of graph invariants. Through examples involving planar graphs, embedded graphs, and graph polynomials, participants will develop a deeper understanding of the rich interplay between combinatorics and topology.

Prerequisites

Participants should have a basic background in graph theory, including graphs, paths, cycles, connectivity, and elementary combinatorial concepts. Familiarity with graph polynomials is not required.